

7. CIRCLES

* The set of points equidistant from a fixed point in a plane is called circle.

* The fixed point in the circle is called centre (o) and the fixed distance is called radius (r) of the circle.

* A circle divides the plane in which it lies in to 3 sets of points.

- 1) Interior of the circle 2) Exterior of the circle
- 3) On the circle

* The distance between the centre and any point on the circle is called radius (r) of the circle.

* The line passing through any two points on the circle is called secant of the circle.

* The line segment joining any two points on the circle is called chord.

* The chord passing through the centre of circle is called diameter (d).

* The part of a circle between any two points is called arc.

* Diameter is the longest chord of the circle.

* Infinite diameters can be drawn to a circle.

* In the circle infinite number of radii can be drawn.

* The diameter of a circle is twice (2times) its radius.

* The radius of a circle is half of its diameter.

* $d = 2r$ (or) $r = d/2$

^ Half part of a circle is called semicircle.

* The diameter divides a circle into two semi circle

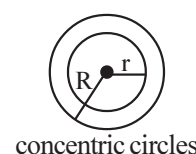
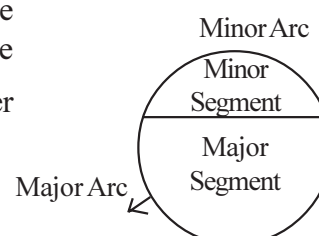
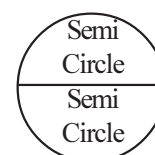
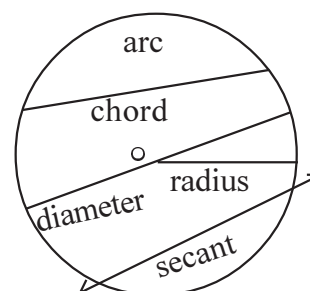
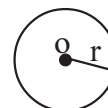
* The chord which does not pass through the centre of circle divides the circle into two segments of circle

* The Bigger part is called major segment and the smaller part is called

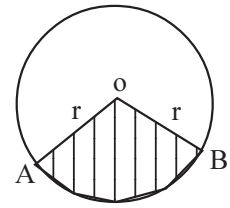
Minor segment..

* The are bounded by major segment is called major arc and the arc of minor segment is called minor arc

* Two or more circles which have same centres and different radii are called concentric circles



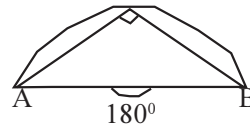
- * The region bounded by arc of a circle and the two radii joining the end points of the arc to the centre of circle is called sector



- * Sum of central angles of the sectors at the centre of a circle is 360°

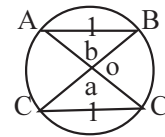
- * The angle at the centre of semi circle is 180°

- * The angle in the semi circle is 90°



- * If the radii of two circles are equal then they are called congruent circles

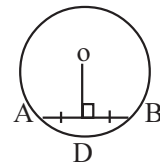
- * Equal chords subtend equal angles at the centre of circle



- * If two chords subtend equal angles at the centre of circle then they are equal

In the adjacent figure , If $AB=CD$ then $\angle a = \angle b$ and if $\angle a = \angle b$ then $AB=CD$

- * The perpendicular drawn from the centre to the chord of circle bisects the chord . If $\overline{OD} \perp \overline{AB}$ then $AD=BD$



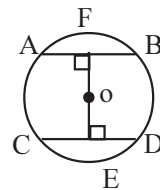
- * If a line drawn from centre of the circle to the chord bisects it then its is perpendicular to the chord.

If $AD=BD$ then $\overline{OD} \perp \overline{AB}$

A circle can be drawn through given 3 non-collinear points in a plane

- * A circle can't be drawn through 3 collinear points.

- * The chords which are equidistant from the centre of circle are equal. If $OE = OF$ then $AB=CD$.



- * If two chords of a circle are equal then they are equidistant from the centre of circle

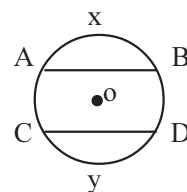
If $AB=CD$ then $OE=OF$

- * If two arcs of a circle are equal then their corresponding chords are also equal.

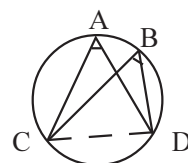
- * If two chords of a circle are equal, then their corresponding arcs are also equal.

If $\widehat{AXB} = \widehat{CYD}$ then $AB = CD$ and If $AB=CD$ then

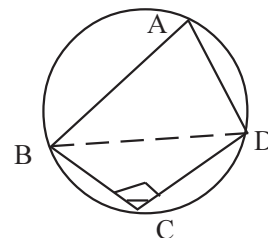
$\widehat{AXB} = \widehat{CYD}$



- * The angles in the same segment of circle are equal . If $\angle A$ and $\angle B$ are angle in the same. Segment of circle.
 $\therefore \angle A = \angle B$

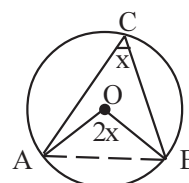


- * The angle formed in major segment is acute angle and the angle formed in minor segment is obtuse angle. $\angle A$ is acute angle formed in major segment $\widehat{BAD}$.



- * The angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at any point on the remaining circle.

$$\therefore \angle AOB = 2 \angle ACB \Rightarrow \angle ACB = 1/2 \angle AOB$$



Solved problems

1. Find the length of chord which is at a distance of 5cm from centre of circle of radius 13cm

Sol: Draw $\overline{OD} \perp \overline{AB}$ OA = 13 cm, OD = 5 cm

$\overline{OD}$ bisects $\overline{AB}$. Let AD = BD = X

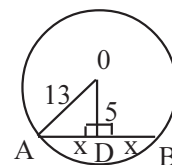
In $\triangle AOD$, $AO^2 = AD^2 + OD^2$ ($\therefore$ Pythagoras theorem)

$$13^2 = x^2 + 5^2 \Rightarrow x^2 = 13^2 - 5^2 = 169 - 25 = 144$$

$$x = \sqrt{144} = 12$$

$$AB = AD + BD = 12 + 12 = 24 \text{ cm}$$

Length of the chord = 24cm



2. In the adjacent figure O is centre of the circle. Its radius is 5 cm.

$\overline{OP} \perp \overline{AB}$, $\overline{OQ} \perp \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$

If CD = 8cm and AB = 6 cm then find PQ ?

Solve: $\overline{OP} \perp \overline{AB}$ and $\overline{OQ} \perp \overline{CD}$

$\therefore$ P, Q are the mid points of AB and CD

$$AP = 6/2 = 3 \text{ cm}, \quad CQ = 8/2 = 4 \text{ cm}$$

In $\triangle APO$ and $\triangle COQ$, AO = CO = 5 (radius)

$$AO^2 = AP^2 + PO^2 \quad CO^2 = CQ^2 + QO^2$$

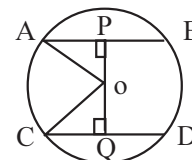
$$5^2 = 3^2 + PO^2 \quad 5^2 = 4^2 + QO^2$$

$$PO^2 = 25 - 9 \quad QO^2 = 25 - 16$$

$$= 16 \quad = 9$$

$$PO = \sqrt{16} = 4 \text{ cm} \quad QO = \sqrt{9} = 3 \text{ cm}$$

$$\therefore PQ = 4 + 3 = 7 \text{ cm}$$



3. An equilateral triangle is inscribed in a circle. The side of the triangle is 9cm. Find its area?

Solve : Draw medium AB. It will be altitude and perpendicular bisector of AB

$$AG : GD = 2 : 1 \quad BD = CD = 9/2 = 4.5 \text{ cm}$$

$$\text{In } \triangle ABD, AB^2 = AD^2 + BD^2$$

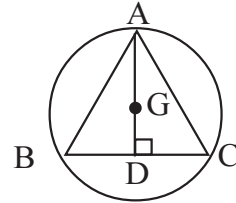
$$AD^2 = AB^2 - BD^2 = 9^2 - (9/2)^2 = 81 - 81/4 = 324 - 81/4 = 243/4$$

$$AD = \sqrt{\frac{243}{4}} = \frac{9\sqrt{3}}{2} \text{ cm}$$

$$AG = \frac{2}{3} \times AD = \frac{2}{3} \times \frac{9\sqrt{3}}{2} = 3\sqrt{3} \text{ cm}$$

$$\therefore \text{Radius of the circle} = 3\sqrt{3} \text{ cm}$$

$$\text{Area of the circle} = \pi r^2 = \frac{22}{7} \times (3\sqrt{3})^2 = \frac{22}{7} \times 27 = 84.8 \text{ cm}^2$$



4. In the adjacent figure 'O' is the centre of circle, AB is a chord which is not a diameter. find $\angle ADB$

$$\text{Sol : } \angle BPC + 110^\circ = 180^\circ$$

(straight angle)

$$\angle BPC = 180 - 110 = 70^\circ$$

$$\text{in } \triangle BPC, \angle BCP + 25 + 70 = 180^\circ$$

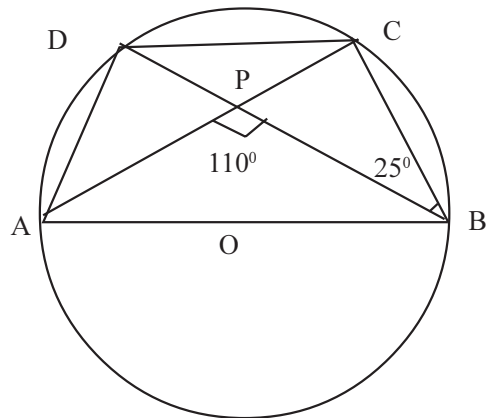
(angle sum property of triangle)

$$\angle BCP = 180 - 95 = 85^\circ$$

$$\text{But } \angle ACB = \angle ADB$$

(angles in the same segment of circle)

$$\therefore \angle ADB = 85^\circ \quad (\because \angle ACB = \angle BCP)$$



5. In the given figure O is centre of circle. $\overline{AC}$ and $\overline{BD}$ are two chords which intersect perpendicularly

at E. If $\angle OAB = 25^\circ$ then find $\angle EBC$

Sol : $OA = OB$ (Radii of circle)

$$\angle OAB = \angle OBA = 25^\circ (\because OA = OB)$$

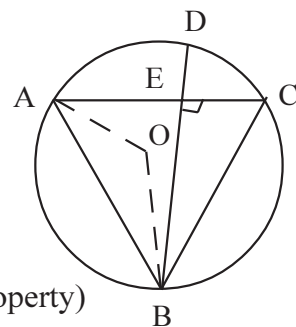
$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

(angle sum property of D)

$$\angle AOB = 180 - (25 + 25) = 180 - 50 = 130^\circ$$

$$\text{In } \triangle EBC, \angle EBC + 65 + 90 = 180^\circ \text{ (angle sum property)}$$

$$\angle EBC = 180 - (90 + 65) = 180 - 155 = 25^\circ$$



6. From the adjacent figure find the values of

i) $\angle BOC$ ii) $\angle AOC$

$OA = OB = OC$ (Radii of circles)

$\angle OCB = \angle OBC = 55^\circ$ ($\because OC = OB$)

$\angle BOC + \angle OCB + \angle OBC = 180^\circ$

(angle sum property of $\triangle OBC$)

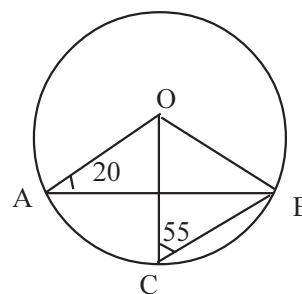
$\angle BOC = 180 - (55 + 55) = 180 - 110 = 70^\circ$

$\angle OAB = \angle OBA = 20^\circ$ ($OA = OB$)

$\angle AOB + \angle OAB + \angle OBA = 180^\circ$ (angle sum property)

$\angle AOB = 180 - (20 + 20) = 180 - 40 = 140^\circ$

$\angle AOC = \angle AOB - \angle BOC = 140 - 70 = 70^\circ$



7. In a circle of centre O, $\overline{AB}$ and $\overline{CD}$ are two equal chords which intersect at E. M, N are the mid points of $\overline{AB}$ and $\overline{CD}$ respectively then what is OMEN?

Sol: M, N are mid points of $\overline{AB}, \overline{CD}$

$\therefore \overline{OM} \perp \overline{AB}$ and $\overline{ON} \perp \overline{CD}$

$OM = ON$ (Equal chords are equidistant from the centre)

In $\triangle OME, \triangle ONE$, $OM = ON$

$\angle OME = \angle ONE = 90^\circ$

$OE = OE$ (common side)

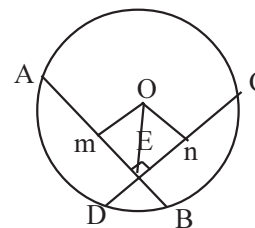
ACC to RHS congruency rule $\triangle OME \cong \triangle ONE$

$ME = NE$ (CPCT)

$OM = ON, ME = NE$ and $\angle ONE = 90^\circ$

Opposite sides are equal and one angle is right angle

$\therefore$ OMEN is a rectangle



8. In the circle with centre 'O' $\overline{AB}, \overline{CD}$ are two diameters. Find the value of $\angle AOC$

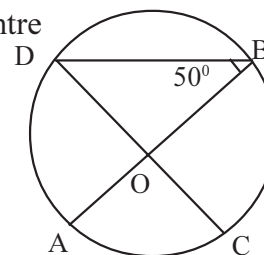
Sol: The arc AD subtends an angle of 50° at B

$\angle AOD$ is the angle subtended by same arc at the centre

$\therefore \angle AOD = 2 \angle ABD = 2 \times 50 = 100^\circ$

But $\angle AOD + \angle AOC = 180^\circ$ (linear pair)

$\therefore \angle AOC = 180 - 100 = 80^\circ$



9. The radii of the circles of centres A and B are 5cm and 3cm respectively. They intersect at P and Q.

If $PQ = 4$ cm then length of $\overline{AB}$ is

Sol : $\overline{AB}$ is perpendicular bisector of PQ .

Join AB, BP

$AP = 5$ m , $BP = 3$ cm (radii) , $PQ = 4$ cm

$PC = CQ = 4/2 = 2$ cm

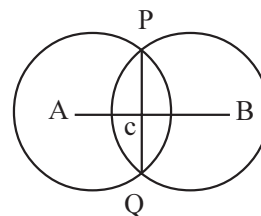
In ΔPAC , $PA^2 = PC^2 + AC^2$

$$\Rightarrow 5^2 = 2^2 + AC^2 \Rightarrow AC^2 = 25 - 4 = 21 \Rightarrow AC = \sqrt{21}$$

In ΔPBC , $PB^2 = PC^2 + BC^2 \Rightarrow 3^2 = 2^2 + BC^2$

$$\Rightarrow BC^2 = 9 - 4 = 5 \Rightarrow BC = \sqrt{5}$$

$$AB = (\sqrt{21} + \sqrt{5}) \text{ cm}$$



10. In the circle of centre A. ABCD is a square.

If $BD = 4$ cm find radius of the circle?

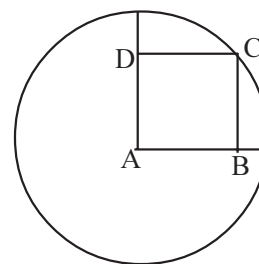
Sol : ABCD is a square

$\overline{AC}, \overline{BD}$ are its diagonals

$BD = 4$ cm $\therefore AC = 4$ cm ($AC = BD$)

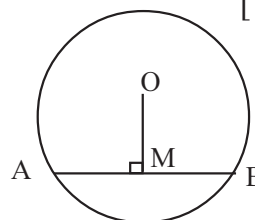
But AC is radius of circle

$\therefore$ Radius = 4cm



LEVEL -1

- Which determines the congruency of two circles []
 a) Their radii b) Their chords c) Their arcs d) All the above
- $\overline{AB}$ is a chord of a circle with center O. $\overline{OM}$ is line drawn from O to the mid point of $\overline{AB}$ then []
 a) $AM = BM$ b) $\overline{OM} \perp \overline{AB}$ c) A and B d) None of these
- In the adjacent figure O is the centre of circle. $OM = 3$ cm, $AB = 8$ cm then radius of the circle is []
 a) 4 cm b) 5 cm
 c) 3 cm d) 7 cm

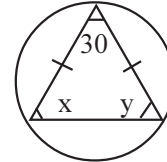


4. In a circle of centre O. $\overline{OM}$ and $\overline{ON}$ are the perpendiculars drawn to the chords $\overline{PQ}$ and $\overline{RS}$ respectively. If $OM = ON$ and $PQ = 6\text{ cm}$ then length of $\overline{RS}$ is

a) 6 cm b) 3 cm c) 9 cm d) 4.5 cm

5. From the adjacent figure the value of x and y are

a) $x = y = 75^\circ$ b) $x = 75^\circ, y = 75^\circ$
c) $x + y = 150^\circ$ d) all the above



6. If the chords $\overline{AC}$ and $\overline{AD}$ are equidistant from the centre of the circle then

a) $AC = AD$ b) $AC \neq AD$
c) $AC = 2 AD$ d) $AD = 2 AC$

7. The region bounded by an arc and the two radii joining the end points of the arc is called []

a) Circle b) Sector c) Arc d) Chord

8. Which of the following is true? []

a) The angle in the major segment is acute
b) The angle in the minor segment is acute
c) The angle in the major segment is obtuse
d) The angle in the semicircle is straight angle

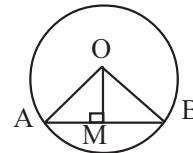
9. In a circle if the distance between centre to the chord $\overline{AB}$ is more than the distance between centre to chord $\overline{CD}$ then []

a) $AB = CD$ b) $AB > CD$ c) $AB < CD$ d) $CD < AB$

10. In the adjacent figure radius of circle is 2 cm ;

If $OM = 1\text{ cm}$ then $AB = \dots \text{ cm}$

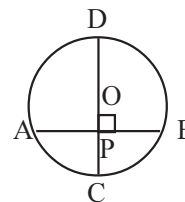
a) $\sqrt{3}$ b) $2\sqrt{3}$
c) 2 d) 3



11. In the circle of centre O, $\overline{AB}$ is chord.

$\overline{CD}$ is diameter and $\overline{CD} \perp \overline{AB}$ then $BP = \dots$

a) CP b) AP
c) OC d) OD

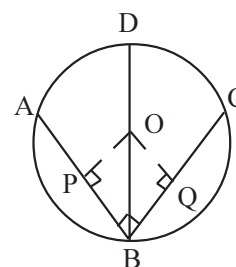


12. In the adjacent figure O is centre of the circle.

$\overline{AB}, \overline{BC}$ are two chords. $\overline{BD}$ is its diameter.

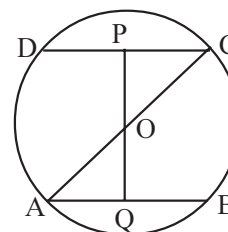
If $\overline{BD}$ bisects $\angle ABC$ then $AB = \dots\dots\dots$

- a) BC b) BD
c) $BO + DO$ d) $2 BC$



13. In a circle of centre O, $\overline{AB}$ and $\overline{CD}$ are parallel chords. $\overline{AC}$ is diameter then $AB = \dots\dots\dots$

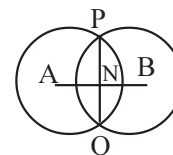
- a) CD b) OA
c) AC d) OC



14. The circles with centres A and B intersect at

P and Q. If $\overline{AB}$ and $\overline{PQ}$ intersect at M then $\angle PMA =$

- a) 60° b) 180°
c) 45° d) 90°



15. The numebr of circles can be drawn through one point in a plane is
[]

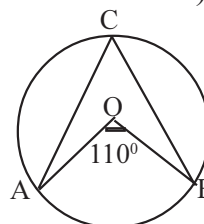
- a) 1 b) Can't draw c) Infinite d) O

16. If a chord subtends a right angle at any point on the remaining circle then its is
[]

- a) Circle b) Semi circle c) Major segment d) Minor segment

17. The relation between the angle subtended by an arc at the centre of circle and the angle subtended by the same arc at any point on the remaining circle is
[]

- a) Equal b) 2 times c) 3 times d) 4 times



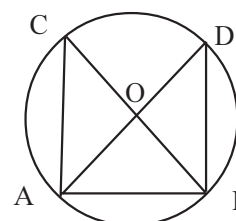
18. In the adjacent figure, $\angle ACB = \dots\dots\dots$

- a) 55° b) 110°
c) 220° d) 70°

19. In the adjanet figure 'O' is centre of circle and

$\angle ACB = 60^\circ$ then $\angle ADB = \dots\dots\dots$

- a) 30° b) 120°
c) 90° d) 60°



20. A triangle is inscribed in a semi circle. If the smallest angle of that triangle is 40° .

The remaining angles are []

- a) $100^\circ, 40^\circ$ b) $60^\circ, 80^\circ$ c) $90^\circ, 50^\circ$ d) $120^\circ, 20^\circ$

LEVEL-2

1. In a circle $\widehat{AXB} = \widehat{AYB} = 180^\circ$ and the length of $\widehat{AXB} = 44\text{cm}$

then the length of $\widehat{AB}$ is cm []

- a) 14 b) 28 c) 21 d) 7

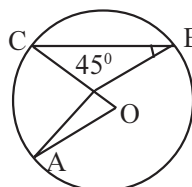
2. The angles in the same segment of circle are $2x + 10$ and $x + 45$. Find x []

- a) $55/2$ b) $35/2$ c) 35 d) 55

3. In the adjacent figure $\angle ABC = 45^\circ$,

$\angle AOC = \dots\dots\dots$

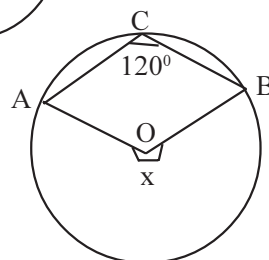
- a) 90° b) 135°
c) 80° d) 55°



4. In the adjacent figure O is centre of circle.

If $\angle ACB = 120^\circ$, then value of x is

- a) 120° b) 60°
c) 240° d) 30°

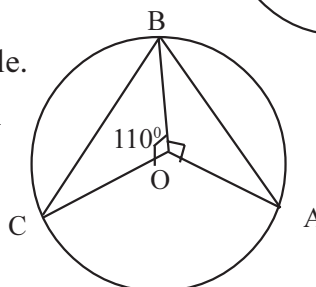


5. In the adjacent figure 'O' is centre of circle.

If $\angle AOB = 90^\circ$ and $\angle BOC = 110^\circ$ then

$\angle ABC =$

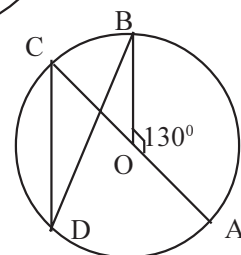
- a) 160° b) 80°
c) 180° d) 45°



6. In the circle of centre O, AC is diameter.

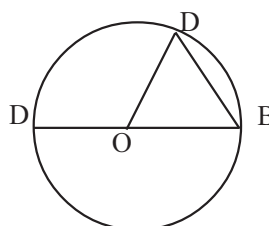
If $\angle AOB = 130^\circ$ then $\angle BDC = \dots\dots\dots$

- a) 50° b) 55°
c) 65° d) 25°



7. In the adjacent figure 'O' is centre of circle and $OC = BC$ then $\angle BOC = \dots\dots\dots$

- a) 120° b) 240°
c) 180° d) 60°



- a) $AB^2 + BC^2 = AC^2$ b) $AB^2 + AC^2 = BC^2$
c) $AC^2 + BC^2 = AB^2$ d) All the above

- The value of $\angle ABC$ is

-

- a) $AC = BD$ b) $A + B = C + D$
c) $BC = AD$ d) $AB = CD$

- $\angle AEB = x, \angle AFB = y, \angle AOB = z$ then the relation between x, y and z is.....

-

- a) 6m b) 5m c) 9.6 m d) 6. 9 m

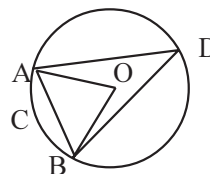
- a) $20\sqrt{3}$ m b) $10\sqrt{3}$ m c) $40\sqrt{3}$ m d) 20 m

- $\angle BOC = 30^\circ$, $\angle BOA = 60^\circ$ and D is a point on the remaining circle of $\widehat{ABC}$ then $\angle ADC = \dots\dots\dots$

-

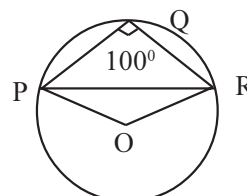
15. In the adjacent figure length of the chord AB is equal to the radius of circle. The angle subtend by the arc in major segment and minor segment respectively are

- a) 150° , 30° b) 30° , 150°
c) 60° , 30° d) 30° , 60°



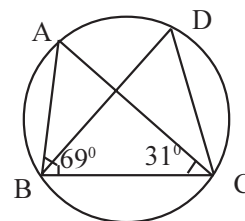
16. In the adjacent figure $\angle PQR = 100^\circ$, P,Q,R are 3 points on the circle with centre O. then $\angle OPR =$

- a) 10° b) 20°
c) 100° d) 40°



17. In the given figure, $\angle ABC = 69^\circ$ and $\angle ACB = 31^\circ$ then $\angle BDC = \dots$ []

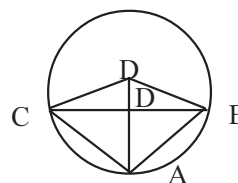
- a) 69° b) 31°
c) 100° d) 80°



18. O is the centre of the circle. The length of the chords $AB = AC = 6\text{cm}$. Radius of circle is

5 cm then $BC = \dots\dots\dots$

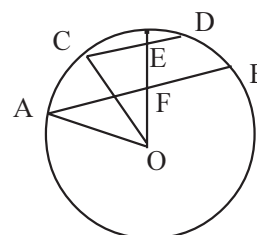
- a) 9.6 cm b) 4.8 cm
c) 19.2 cm d) 8 cm



19. In the adjacent figure $\overline{OE} \perp \overline{CD}$, $\overline{OF} \perp \overline{AB}$, $\overline{AB} \parallel \overline{CD}$

$AB = 48\text{ cm}$ and $CD = 20\text{ cm}$. If radius of the circle is 26cm then length of $\overline{EF}$ is

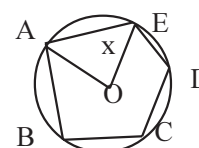
- a) 6 cm b) 8 cm
c) 14 cm d) 16 cm



20. A regular pentagon is inscribed in a circle with centre O.

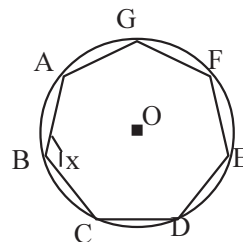
Then value of x is

- a) 36° b) 54°
c) 72° d) 108°



21. In a circle 7 equal chords are given then $x = \dots\dots$
(O is centre of circle)

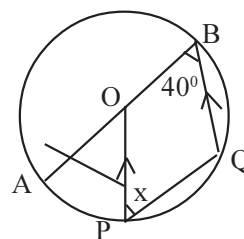
- a) $128\frac{4}{7}^0$ b) $180\frac{6}{7}^0$
c) $128\frac{1}{7}^0$ d) $360\frac{4}{7}^0$



22. In the given circle $\overline{AB}$ is diameter. $\overline{OP} \parallel \overline{BQ}$.

$\angle ABQ = 40^0$ then $x = \dots\dots\dots$

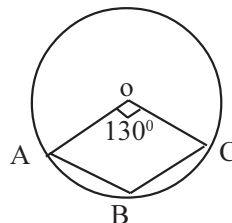
- a) 60^0 b) 70^0
c) 80^0 d) None



23. In the figure O is centre of circle.

If $\angle AOC = 130^0$ then $\angle ABC = \dots\dots\dots$

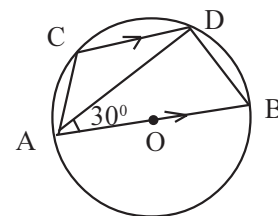
- a) 50^0 b) 65^0
c) 115^0 d) 130^0



24. In the given figure $\overline{AOB}$ is diameter of the circle.

$\overline{AB} \parallel \overline{CD}$ and $\angle BAD = 30^0$ then $\angle CAD = \dots[\quad]$

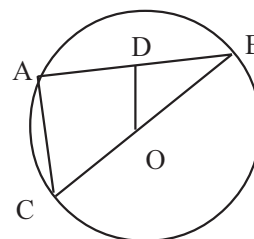
- a) 30^0 b) 60^0
c) 45^0 d) 50^0



25. $\overline{AB}$ is a chord of a circle with centre O. $\overline{OD} \perp \overline{AB}$.

OD = 6cm and AC = 3K then value of K is .

- a) 3 b) 4
c) 5 d) 6



* * *

CIRCLES

KEY

level -1

1	A
2	C
3	B
4	A
5	D
6	A
7	B
8	A
9	C
10	B
11	B
12	A
13	A
14	D
15	C
16	B
17	B
18	A
19	D
20	C

level -2

1	B
2	C
3	A
4	C
5	B
6	C
7	D
8	C
9	B
10	D
11	A
12	C
13	A
14	C
15	B
16	A
17	D
18	A
19	C
20	B
21	A
22	B
23	C
24	A
25	B